Properties of Opportunistic and Collaborative Wireless Mesh Networks

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***Abstract*—While the cost of nodes in a wireless mesh network is decreasing, the price tag of the network as a whole is best minimized by deploying the fewest number of nodes that still achieves some target network connectivity. In this document, we study two techniques to improve the connectivity of the network without increasing the density, and thus the cost, of the network.**

The two techniques are opportunistic routing [12], [13] and collaborative forwarding [11]. Opportunistic routing makes use of links which become temporarily available due to instantaneous radio conditions. Collaborative forwarding forms associations between small connected clusters to forward packets to nodes outside of the range of any node in the cluster.

This document shows that both technique significantly decrease the critical connectivity threshold for the wireless mesh network, and thus provide significant connectivity gain. We focus on analytical techniques to prove the results, and confirm them by performing a numerical evaluation.

***Index Terms*— Wireless mesh networks, collaborative networks, opportunistic networks.**

I. Introduction

Wireless mesh networks include an ever widening array of applications. These applications take the form of wireless metropolitan hotspots [1], [2], [3], [4], [5], [6], to provide broadband connectivity in wide municipal areas [7], [8]; sensor networks [9], to monitor, manage, control or sense a given domain; or peer-to-peer ad hoc networking to establish an impromptu communication between mobile terminals without the support of an infrastructure, for instance in emergency response scenarios [10]. While these applications seem very different, in all cases, the underlying networks have similar characteristics: the topology and the number of nodes are dictated by factors external to the network.

In a wireless mesh networks, the nodes are placed semi-regularly, but the node position depends on the availability of a lamp post, on the ownership of that lamp post, and on the underlying physical geography. In some last mile access networks [1], [2], it is the participants in the network who condition the topology. In a sensor network, nodes might be tossed out of plane, sown in the wind, carried by animals [15] or drifting with the oceanic currents [16]. In an ad hoc network of peers, the participant might be mobile. The number of nodes in some of the examples is set by the price that the network operator is willing to incur, while in other, it is an indication of the popularity of the network. In all cases, the topology is not deterministic, and an increase in the number of nodes is not a practical option.

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A. Critical Densities

A random network topology might not yield a perfectly connected network. The density in some areas of the network might be too low to sustain communication across multiple hops, while in others, it is high enough to carry information from the nodes in the field to the network gateways, be they sensor network sinks or access controller towards the wider internet.

Since in any random distribution of the nodes, there will be some outliers, total connectivity is not a practical target. However, the wireless mesh network topology should ensure that there is a large enough connected component over which the mesh network can communicate. If the connected component encompasses a large fraction of the nodes, then the network can function properly at the cost of only a few nodes being disconnected.

In a milestone paper, Hall [14] proved that, if the nodes are distributed according to a two-dimensional Poisson process, and if two nodes are connected whenever they are within distance r of each other, then there exists a critical density λ_c such that, for all networks with node density $\lambda > \lambda_c$, there exists an infinite cluster, ie. an infinitely large connected component.

This result is primordial, as setting up a network with a sub-critical density will give a disconnected network with probability one. All connected components will have finite size, and the network will be composed only of isolated islands with no bridges in between. On the other hand, for a super-critical density, then there exists an infinite cluster with probability one, and by placing the gateways inside this cluster, all the nodes in the cluster can actively monitor the environment and forward their data stream to the gateways.

Since there is an increasing cost associated with a higher density, the critical value λ_c gives both a lower bound and a target to achieve. One would want to deploy the network at a density above λ_c, but not too far.

B. Contribution and Organization

In this document, we discuss two techniques to virtually increase the density of the network without increasing the node density λ. These techniques are not original to this document and have been introduced elsewhere. However, the benefit in terms of improved network connectivity has not been assessed earlier. The contribution of the paper is to prove analytically and through simulations the gain achieved by these techniques in lowering the connectivity threshold.

The two techniques are:

- Opportunistic scheduling. This was introduced in [12] and an IP-level extension of AODV to support opportunistic routing, the OPRAH protocol, was introduced in [13]. Opportunistic routing forwards packets to nodes based on instantaneous radio conditions, and thus adds temporary links into the network connectivity.
- Collaborative forwarding. This was introduced in [11]. The main idea here is for a cluster of nodes to join forces in order to forward a packet to a node outside of the communication range of all the nodes in the cluster. Each node can only connect to the circle of radius r around them, but a pair of nodes which are $d < r$ apart, can connect to nodes outside of the union of the two circles of radius r and jointly reach nodes further away.

In the next section II, we consider the connectivity gain achieved by the use of collaborative forwarding. In section III, we consider the benefit gained from opportunistic routing. In section IV, we simulate the connectivity threshold for different mesh networks, and present some numerical values.

II. COLLABORATIVE NETWORKS

In the paper *Collaboration Improves the Connectivity of Wireless Networks* [11], the study of the percolation threshold in the case of an extended network [1] is performed using only simulation, and shows that the percolation threshold does seem to appear strictly lower than in the non-collaborative model. In this paper, we complement [11] by proving and quantifying that the percolation threshold is indeed significantly less in the collaborative model than in the non-collaborative case.

We consider here free space propagation, namely a path loss exponent $\alpha = 2$, but other α could be considered as well. We will vary α in the numerical evaluation in Section IV.

Consider a 2 dimensional Poisson process with intensity λ in which two nodes are connected if they are within distance $\rho = 1/\sqrt{\pi}$ of each other in the non-collaborative model (unit disk connectivity). Denote by $G(\rho, \lambda)$ the connectivity graph defined by the non-collaborative model built upon the Poisson process with intensity λ and connectivity radius ρ. Denote by λ_c the critical intensity for such graph $G(\rho, .)$. Define $\epsilon = 0.1$.

Definition II.1: We say that two nodes X, Y can *collaborate* to reach a third node Z if:

- the distance $d_{X,Y} = |X - Y| < \rho$, so that X and Y can communicate directly, and
- Z satisfies:

$$\frac{1}{d_{X,Z}^2} + \frac{1}{d_{Y,Z}^2} > \pi \tag{1}$$

If equation (1) is not satisfied, then Z is out of the reach of the cluster composed by X and Y. More than two nodes could form a larger cluster, but we consider only node pairs collaborating in this document.

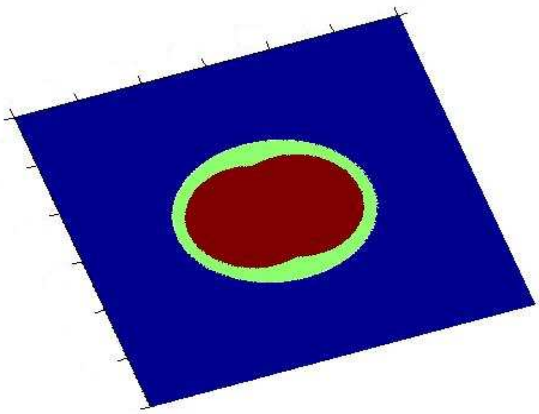

Fig. 1. Coverage Area of Two Nodes Independently and Collaboratively

Figure 1 depicts the area of increased connectivity around two nodes placed at $2/3\rho$ apart: independently, they cover the binocular shaped area in the middle, whereas jointly, they cover the ellipse-looking area [2].

[1] An extended network is a network which increases in size and keeps a constant density as the number of nodes grows

[2] We quickly add that the shape is *not* an ellipse, but can be proven to include one containing both circles

Theorem II.1: Collaboration improves the connectivity at least as much as increasing the intensity λ by a factor $\gamma > 1$.

Proof:

Consider a point X of the Poisson process, and assume it has a neighbor Y at distance $\rho(1+\epsilon)$. This neighbor is not connected in the non-collaborative model. Let's see if it is connected in the collaborative model.

With probability $q = 1 - \exp(-\lambda)$ there exists another point Y in the disk of the connectivity area of X. In the collaborative scenario, X and Y team up to communicate with Z.

The power received at Z by the sum of X and Y collaborating is at least

$$p_Z(\epsilon) = \frac{1}{\rho^2(1+\epsilon)^2} + \frac{1}{\rho^2(2+\epsilon)^2} \quad (2)$$

since Z cannot be further away than $(2+\epsilon)\rho$ from Y.

$p_Z(\epsilon)$ is obviously a decreasing function in ϵ. Since $p_Z(0) = 1.25\pi$, and $p_Z(\infty) = 0$, there exists ϵ^* for which it is equal to π. It is enough for us to observe that $p_Z(0.1) > \pi$, so the signal has enough power to be decoded at Z using X and Y for our chosen value of ϵ.

Since X is now connected to Z with probability that Y exists, and since the probability that Y exists is independent of X and Z due to a property of the Poisson process, and at least equal to $(1-e^{-\lambda})$, this means that we can couple the collaborative graph with a non-collaborative graph $G((1+\epsilon)\rho, \lambda q)$, where q is the probability that Y exists.

The graph $G((1+\epsilon)\rho, q\lambda)$ percolates iff the graph $G(\rho, (1+\epsilon)^2 q\lambda)$ percolates, as it corresponds to a scaling of the Poisson process. It is equivalent to increase the density by a factor $(1+\epsilon)^2$ or increase ρ by $(1+\epsilon)$.

Since

$$\begin{aligned} (1+\epsilon)^2 q &\geq (1+\epsilon)^2(1-e^{-\lambda}) \\ &\geq 1.21(1-e^{-2.195}) \\ &\geq 1.07, \end{aligned} \quad (3)$$

where in the second step, we took the proven lower bound for λ mentioned in [11]. This ensures that the collaborative graph has an infinite cluster whenever $\lambda > \lambda_c/1.07$. Namely this means that the collaboration increases the connectivity as much as increasing the intensity λ by 7%.

Actually, for $\lambda \sim 4.5$ the intensity increase is about 20%. Taking $\gamma = 1.07$ completes the proof of the Theorem. ∎

III. Opportunistic Networks

We now turn our attention to opportunistic routing, which we briefly described in Section I. The improvement of opportunistic routing could be quantified as follows: consider a sender s. The range of this sender is given by the value r. Two nodes are connected if their distance relative to each other is less than r.

In traditional routing, one uses only steady links, ie. such that the packet error rate is below some threshold. If one ignores interferences, and considers only an isotropic propagation, this basically sets a maximum value for r which satisfies the packet error rate.

Consider again a graph in which nodes are distributed according to a Poisson plane distribution with rate λ. For a deterministic connectivity radius R, there is a critical density λ_c such that, for $\lambda > \lambda_c$, the probability that there exists an infinite connected subgraph is $\Theta(\lambda) > 0$. A graph is said to be *percolating* if it includes such an infinite connected subgraph.

Now it is obvious that for another graph built upon the same Poisson distributed points, but such that the connectivity radius is $\tilde{r} \geq R$, then

$$\begin{aligned} \tilde{\lambda}_c &\leq \lambda_c \text{ and} \\ \tilde{\Theta}(\lambda) &\geq \Theta(\lambda). \end{aligned} \quad (4)$$

This states that your connectivity cannot get worst if the range is increased, which is trivial.

What we would like to see true is that the inequalities in (4) are strict inequalities. This would allow us to establish that there is a substantial benefit to the use of opportunistic routing. For instance, in the case of a strict inequality, one could have a system such that the rate of the Poisson distribution of the points in the plane satisfies:

$$\tilde{\lambda}_c < \lambda < \lambda_c \quad (5)$$

This would mean that in a large network thus distributed, illustrated for instance in Figure 2, using a link quality metric to define *a priori* which links are reliable or not, would result in the network in black, with only local islands of connectivity, while using opportunistic routing would result in the network in gray, with infinitely many connected nodes with some positive probability. One would be useless, while the other could perform its function.

A. Critical Intensity of Opportunistic Routing

Given a node x, it is connected to all the nodes within Euclidian distance $r_x(t)$ of x, where $r_x(.)$ is a stationary random process with probability density μ_r at a given time t. We will only consider the connectivity at a single time instance, as our stationarity assumption implies the probability of an infinite cluster to stay constant over time, and thus we omit the index t from now on.

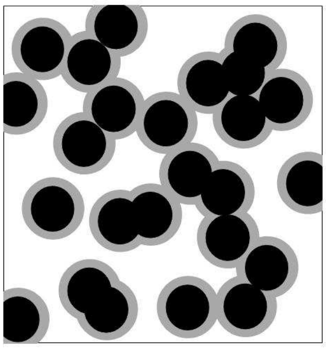

Fig. 2. Opportunistic routing increases the connectivity

We also assume that μ_r is the same for all points, and that μ_r has finite support $(0, r_{max})$. Physical systems do not connect infinitely far apart, so the assumption is reasonable.

Typical routing protocols only consider links for which they can assess some level of confidence. These routing protocols only consider links for which the probability of successful transmission is greater than some θ, say θ =90%. Define by r_d the radius such that:

$$\theta = \int_{r_d}^{r_{max}} \mu_r dr \tag{6}$$

With probability θ, two nodes at a distance less than r_d are connected.

The deterministic routing protocols therefore use the variable:

$$\hat{r} \triangleq \min(r, r_d), \tag{7}$$

which is equal to r_d w.p. θ, to establish whether or not two nodes are connected. Opportunistic routing uses the full value r. We call *"Deterministic Model"* the graph build upon the 2-dimensional Poisson process and the connectivity radius $\hat{r}$. We will denote this model (D). Similarly, we call *"Opportunistic Model"* the graph built upon the same 2-dimensional Poisson process and the connectivity radius r. Similarly, we denote this model with (O).

Define $\Theta(\lambda)$ to be the probability that there exists an infinite connected component for an underlying Poisson process with intensity λ. Θ^d denotes this probability for the deterministic model (D), and similarly, Θ^o for the opportunistic model (O).

Lemma III.1: There exists a value λ_c^d for which:

$$\forall \lambda > \lambda_c^d, \Theta^d(\lambda) = 1 \tag{8}$$

Proof:

It should be obvious that the deterministic routing system will percolate for some critical value λ_c^d. It is a consequence of [14], but one can see it easily by considering two Poisson boolean models (P1) and (P2).

The first one (P1) is built upon the same Poisson distribution, but for which the connectivity radius is fixed and equal to r_d. Namely, each node is connected to neighbors at distance within r_d. This model (P1) is different from our deterministic model, since they coincide only when $r > r_d$, in which case $\hat{r} = r_d$. When $r < r_d$, $\hat{r} = r < r_d$. Since in both cases, $\hat{r} \leq r_d$, the Poisson boolean model (P1) dominates the deterministic model. It is a well know result that the Poisson boolean model percolates for some λ_c^b.

One can bound the deterministic model below by thinning the Poisson process: for each point in the graph, with probability θ it is kept, and with probability $1 - \theta$ it is removed. Thinning the Poisson process gives us another Poisson boolean model (P2), but with intensity now $\lambda\theta$. This model is equivalent to the deterministic model (D) with the links strictly shorter than r_d removed. As such, it is dominated above by the deterministic model, and percolates for λ_c^b/θ.

The two models P1 and P2 combined ensure that there exists a value $0 < \lambda_c^d < \infty$ (actually $\lambda_c^b < \lambda_c^d < \lambda_c^b/\theta$) such that there is a phase transition: for $\lambda < \lambda_c^d$, the largest connected component has finite size with probability one. For $\lambda > \lambda_c^d$ the size of the largest connected component is infinite with probability $\gamma(\lambda) > 0$. ∎

Define λ_c^o to be the critical value for the Opportunistic model (O). As we discussed above, $\lambda_c^o \leq \lambda_c^d$. We can now state the following theorem:

Theorem III.1: There exists a constant $\beta > 1$ such that:

$$\lambda_c^o \leq \frac{\lambda_c^d}{\beta} \tag{9}$$

This states that there is a huge gain from opportunistic routing when the rate λ of the underlying Poisson process is between λ_c^d/β and λ_c^d, as in the deterministic case, the network will be disconnected with probability one. In the opportunistic case, the network will have an infinite component with some positive probability.

Proof: Consider the opportunistic model with rate λ such that:

$$\frac{\lambda_c^d}{\beta} < \lambda < \lambda_c^d \tag{10}$$

where $\beta > 1$ will be made explicit later.

The expected number of neighbors connected to a given node in the deterministic model (D) is given by:

$$E[\text{\# of neighbors in (D)}]$$

$$
\begin{aligned}
&= \int_0^{r_{max}} \pi\lambda\hat{r}^2\mu_r dr \\
&= \lambda\pi\left[\theta r_d^2 + \int_0^{r_d} r^2\mu_r dr\right] \\
&= \Gamma_D(\lambda) \qquad (11)
\end{aligned}
$$

The number of neighbors for the opportunistic model (O) can be computed similarly, and is given by $\Gamma_O(\lambda)$:

$$
\begin{aligned}
\Gamma_O(\lambda) &= \lambda\pi\int_0^{r_{max}} r^2\mu_r dr \\
&= \Gamma_D(\lambda) + \lambda\pi\int_{r_d}^{r_{max}} (r^2 - r_d^2)\mu_r dr. \qquad (12)
\end{aligned}
$$

Denote by

$$
\begin{aligned}
\Delta\Gamma(\lambda) &= \Gamma_O(\lambda) - \Gamma_D(\lambda) \\
&= \lambda\pi\int_{r_d}^{r_{max}} (r^2 - r_d^2)\mu_r dr. \qquad (13)
\end{aligned}
$$

We now define $\beta = 1 + \frac{\Delta\Gamma(\lambda)}{\Gamma_D(\lambda)}$ and consider another deterministic model (D2) with the same connectivity rule as the deterministic model (D) but intensity $\beta\lambda$ for the underlying Poisson process.

First, we note that by our choice of λ in equation (10), we have $\lambda_c^d < \beta\lambda$ and our model (D2) is thus supercritical. As such, there exists an infinite component in the model (D2) with probability one.

Also, the number of neighbors of a node is given by, since $\Gamma_D(\lambda)$ is linear in λ:

$$
\begin{aligned}
\Gamma_D(\beta\lambda) &= \beta\Gamma_D(\lambda) = \Gamma_D(\lambda) + \Delta\Gamma(\lambda) \\
&= \Gamma_O(\lambda) \qquad (14)
\end{aligned}
$$

Thus a node in (D2) has as many neighbors as a node in (O), which implies that one can map an infinite path in (D2) to an infinite path in (O). This means that (0) also has an infinite component.

Since we have proved that there is an infinite component in (O) for any λ which satisfies $\lambda_c^d/\beta < \lambda < \lambda_c^d$, this means that the critical intensity for (0), λ_c^o satisfies:

$$
\lambda_c^o \leq \frac{\lambda_c^d}{\beta}, \qquad (15)
$$

which completes the proof of the theorem. ▮.

IV. Numerical Evaluation

We estimate numerically the gain that we analytically discussed in the previous sections for opportunistic forwarding and collaborative forwarding.

A. *Opportunistic Simulations*

Since the critical value for the density pertains to an infinite cluster, and thus an infinite network, we cannot assess it by simulation. We approximate by looking at the critical value for which there exist a path connecting two sides of a square of varying size. We consider a square area of side $a = 500m, 700m, 900m$ respectively, and look at the probability of a transversal path joining a node at $(0, a/2)$ to $(a, a/2)$. Since the size of the area is finite, the probability of a crossing path is not a 0-1 law, but it has a sharp threshold which approximates the critical value we are looking for. Also, due to the finiteness of the graph, the probability of a path for opportunistic connectivity only converges to 1 for larger intensities. The threshold becomes sharp when looking at the largest component instead of a single path.

In Figure 3, we plot the benefit for the critical value of the density for the opportunistic forwarding. We consider an opportunistic routing which connects with a node at distance r for two different laws: Rayleigh distribution and uniform distribution. We compare with a static law. In practice, the static law would only keep links with a good enough reliability, and ignore the other links. This means that the opportunistic forwarding has a *wider* connectivity. However, in our simulations, we set the average value of the random connectivity equal to that of the static connectivity. We set this value to 80m.

We see that both random connectivity have a critical density which is earlier than static routing. For Rayleigh distribution, the critical density is approximately at 150 nodes/km^2, 200 nodes/km^2 for the uniform and 225 nodes/km^2.

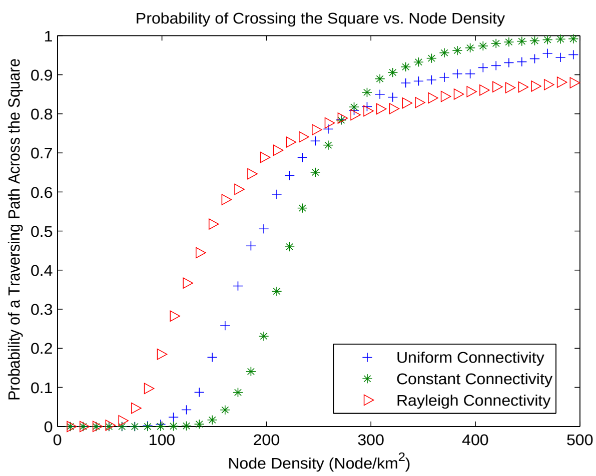

Fig. 3. Critical Density for Opportunistic Routing

In figure 4, we see the impact of the square size on the critical value and see that the behavior of the critical intensity we are considering is independent of the square size, and thus a good approximation for an infinite graph.

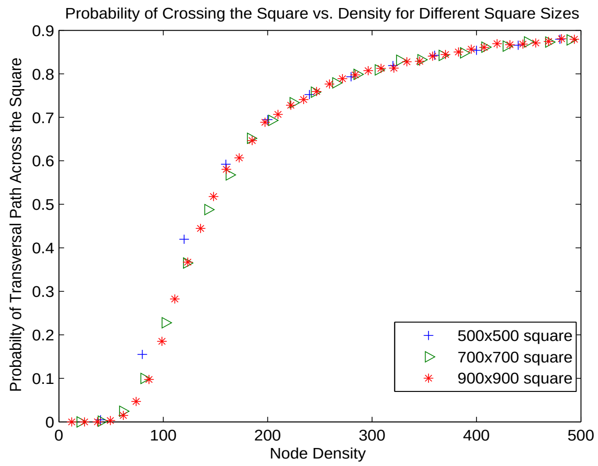

Fig. 4. Connectivity Threshold for Different Square Sizes

B. Collaborative Simulations

In figure 5, we look at the gain for the critical density for collaborative routing. The critical density is about 175 for the collaborative forwarding, and 230 for the non-collaborative forwarding. This means that 175 collaborative nodes can cover the same area as 230 non-collaborative nodes, a gain of about 25 %, larger than the analytical value of the lower bound in Section II.

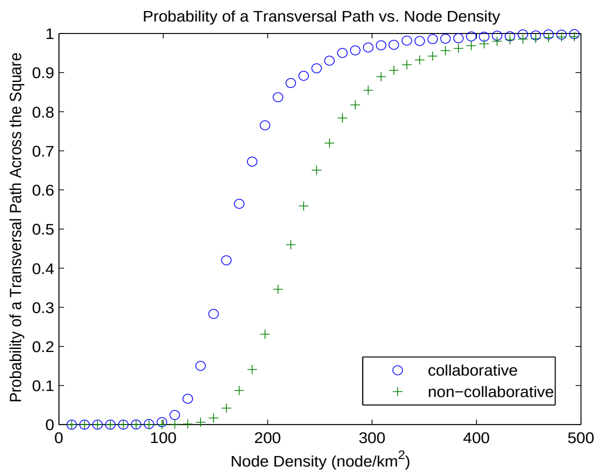

Fig. 5. Connectivity Threshold for Collaborative Routing

The benefit of collaborative routing is dependent on the free space propagation exponent α, as we mentioned in Section II. We plot in Figure 6 the variation of the critical connectivity threshold for different values of α ranging between 2 and 5. We kept the connectivity area of a single node identical for all values of α, namely it is a circle of unit area. Similarly to Section II, $\rho = 1/\pi^{1/\alpha}$ and Equation 1 becomes:

$$\frac{1}{d_{X,Z}^{\alpha}} + \frac{1}{d_{Y,Z}^{\alpha}} > \pi \qquad (16)$$

As we can see, the higher the exponent, the less benefit there is from the collaboration. It is not surprising, as increasing the exponent will make the connectivity pattern look more and more like a step function with a sharp threshold at the connectivity radius, for $d_{X,Z} > 1$ or $d_{Y,Z} > 1$. The power received outside of the connection range will decrease more and more, and in the limit, the connectivity of the collaboration scheme deteriorates to that of the non-collaborative scheme.

However, for practical values of α (for instance, [17] estimates its value in a typical urban environment as being between 3 and 4), one can still see an obvious benefit from collaboration.

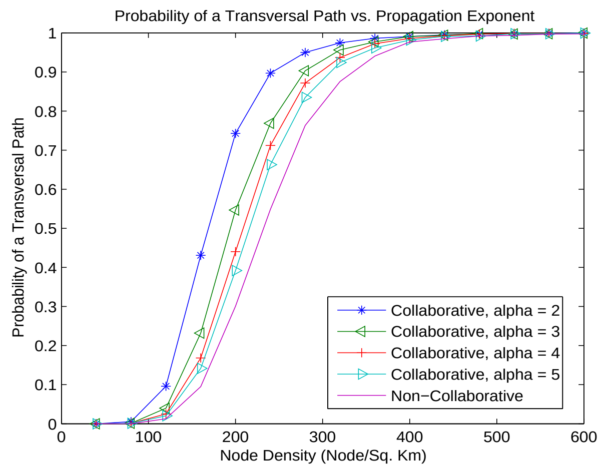

Fig. 6. Connectivity Threshold for Collaborative Routing when Varying the Propagation Exponent

V. Conclusion

We analyzed and evaluated the critical intensity for two existing forwarding schemes in wireless mesh networks: an opportunistic forwarding scheme, and a collaborative forwarding scheme. We have shown formally that both schemes strictly improve on the connectivity of a graph, and reduce substantially the critical threshold required to obtain the existence of an infinite component in particular.

We have supported the analysis by some numerical evaluation. Future research would focus on the design of practical protocols to leverage the benefits analyzed here into actual improvements in the deployed networks. In terms of analysis, the results focus on the existence of an infinite components, which makes the assumption of an infinitely large network. A quantity of interest for a network of finite size N is the fraction of nodes connected into the largest cluster. We would like to study the asymptotic behavior of this quantity as N increases.

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